* Chapter Four *
* Interval Estimation Theory *

Confidence Intervals for population Mean (μ)

The Point estimator for population mean μ is $\bar{x}$ as best estimator, which do not expected to be equal to the parameter μ.

$\bar{x}$ can be used to form the CI for μ depending on the sampling dist. of $\bar{x}$

1_ Building a Confidence Interval for μ when σ^2 is known.

We know that $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ is distributed as a standard normal (z), so we have a 95% confidence interval for the Population Mean μ.

$$P\left(\bar{x} - 1.96\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + 1.96\left(\frac{\sigma}{\sqrt{n}}\right)\right) = 1-\alpha$$

In general for Z_α the $(1-\alpha)$% CI of μ will be as:-

$$P\left(\bar{x} - Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right)\right) = 1-\alpha.$$

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* Z_α values according to α *

α	$1-\alpha$	$(1-\alpha)\%$	Z_α
0.10	0.90	90%	1.64
0.05	0.95	95%	1.96
0.01	0.99	99%	2.58

Ex:- Suppose a sample of 25 students at a University has Sample mean of 127. if the Population Standard deviation is 5.4, calculate the 95% CI For the Population mean.

Sol:-

From table 95% CI $\longrightarrow Z_\alpha = 1.96$

$\bar{x} = 127$, $\sigma = 5.4$, $n = 25$

$$\rightarrow P\left(\bar{x} - Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right)\right) = 1-\alpha$$

$$\rightarrow \left(127 - 1.96\left(\frac{5.4}{\sqrt{25}}\right) < \mu < 127 + 1.96\left(\frac{5.4}{\sqrt{25}}\right)\right)$$

$$\rightarrow 127 - 2.12 < \mu < 127 + 2.12$$

$$\rightarrow 124.88 < \mu < 129.12$$

we are 95% Certain that the Population mean is 124.88 and 129.12

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Ex:- Suppose a sample of 25 Students at a University has Sample mean of 127, if the Population Standard deviation is 5.4, calculate 99% CI for the Population mean.

Sol:- from table 99% CI is 2.58

$n=25, \quad \bar{x}=127, \quad \sigma=5.4$

$$P\left(\bar{X}-Z_{\alpha}\left(\frac{\sigma}{\sqrt{n}}\right)<\mu<\bar{X}+Z_{\alpha}\left(\frac{\sigma}{\sqrt{n}}\right)\right)=1-\alpha$$

$$\left(127-2.58\left(\frac{5.4}{\sqrt{25}}\right)<\mu<127+2.58\left(\frac{5.4}{\sqrt{25}}\right)\right)$$

$$\rightarrow \quad 124.22<\mu<129.78$$

H.W: let $x \sim N(\mu,4)$, $n=16$, $\bar{x}=12$, Find 90% C for μ.