

Chapter Four Interval Estimation Theory

6 - Confidence intervals for the binomial Proportion Parameter p .

- The CI For the Proportion p (Probability for Succeed in binomial Trials), can be given as:

$$p - z \sqrt{\frac{p(1-p)}{n}} < p < p + z \sqrt{\frac{p(1-p)}{n}}$$

Ex: - Consider a random Sample of 144 Families, 48 have 2 or more Cars. Compute the 95% CI for the Population proportion of families with 2 or mor cars.

Sol: $n = 144$, $p = \frac{48}{144} = \frac{1}{3}$

From Table \rightarrow 95% CI $\rightarrow z_{\alpha} = 1.96$

$$1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

\downarrow general Eq.

$$p - z_{\alpha} \sqrt{\frac{p(1-p)}{n}} < p < p + z_{\alpha} \sqrt{\frac{p(1-p)}{n}}$$

$$\frac{1}{3} - (1.96) \cdot \sqrt{\frac{\frac{1}{3} \cdot (\frac{2}{3})}{144}} < p < \frac{1}{3} + (1.96) \sqrt{\frac{(\frac{1}{3})(\frac{2}{3})}{144}}$$

$$0.333 - 0.077 < p < 0.333 + 0.077$$

$$0.256 < p < 0.410$$

* Interval Estimation theory *

Ex2) A random sample of 100 people shows that 25 are left-handed from a 95% CI for the True Proportion of left-handers.

Sol) $n=100$ $p = \frac{25}{100} = 0.25$

$1-p = 1 - \frac{1}{4} = \frac{3}{4}$, 95% $\rightarrow z_{\alpha} = 1.96$

by using general Eq:-

$$p - z_{\alpha} \sqrt{\frac{p(1-p)}{n}} < p < p + z_{\alpha} \sqrt{\frac{p(1-p)}{n}}$$

$$\Rightarrow \frac{1}{4} - (1.96) \sqrt{\frac{\frac{1}{4} \cdot (\frac{3}{4})}{100}} < p < \frac{1}{4} + (1.96) \sqrt{\frac{\frac{1}{4} \cdot (\frac{3}{4})}{100}}$$

$$\Rightarrow 0.1651 < p < 0.3349$$