Chapter Four

Interval Estimation Theory

* Z_α Values according to α *

α	$1-\alpha$	$(1-\alpha)\%$	Z_α
0.10	0.90	90%	1.64
0.05	0.95	95%	1.96
0.01	0.99	99%	2.58

Ex:- Suppose a sample of 25 students at a University has Sample mean of 127. if the Population standard deviation is 5.4, calculate the 95% CI for the Population mean.

$1-\alpha = 95\%$

Sol:

From table 95% CI $\longrightarrow Z_\alpha = 1.96$

$\bar{X} = 127$, $\sigma = 5.4$, $n = 25$

$$\rightarrow P\left(\bar{X} - Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right)\right) = 1-\alpha$$

$$\rightarrow \left(127 - 1.96\left(\frac{5.4}{\sqrt{25}}\right) < \mu < 127 + 1.96\left(\frac{5.4}{\sqrt{25}}\right)\right)$$

$$\rightarrow 127 - 2.12 < \mu < 127 + 2.12$$

$$\rightarrow 124.88 < \mu < 129.12$$

We are 95% certain that the Population mean is 124.88 and 129.12

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Ex:- Suppose a sample of 25 Students at a University has Sample mean of 127, if the Population Standard deviation is 5.4, calculate 99% CI for the Population mean.

Sol:- from table 99% CI is 2.58

$n=25$, $\bar{x}=127$, $\sigma=5.4$

$$P\left(\bar{x}-Z_{\alpha}\left(\frac{\sigma}{\sqrt{n}}\right)<\mu<\bar{x}+Z_{\alpha}\left(\frac{\sigma}{\sqrt{n}}\right)\right)=1-\alpha$$

$$\left(127-2.58\left(\frac{5.4}{\sqrt{25}}\right)<\mu<127+2.58\left(\frac{5.4}{\sqrt{25}}\right)\right)$$

$$\rightarrow \quad 124.22<\mu<129.78$$

H.W let $X \sim N(\mu,4)$, $n=16$, $\bar{x}=12$, Find 90% CI for μ.

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2- Confidence interval for μ when σ^2 is unknown

We use the next best thing, the sample standard deviation S. But with S, insted of a Z distribution we have a T distribution (with (n-1) degrees of Freedom). So

$$P\left(\bar{X} - Z_{\alpha}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + Z_{\alpha}\left(\frac{\sigma}{\sqrt{n}}\right)\right) = 1-\alpha$$

becomes

$$P\left(\bar{X} - t_{n-1}\left(\frac{S}{\sqrt{n}}\right) < \mu < \bar{X} + t_{n-1}\left(\frac{S}{\sqrt{n}}\right)\right) = 1-\alpha$$

Ex:- let $\bar{X} = 74$, $S^2 = 132.7$, $n = 4$, Calculate the 95% CI for μ.

$\boxed{\text{Sol}}$:- $\bar{X} = 74$ $\quad S^2 = 132.7 \rightarrow S = 11.5$

$n = 4$ then degrees of freedom $\rightarrow n-1 = 3$

So from table of t 95% CI is

$t(3, 0.95) = 2.353$

$$P\left(\bar{X} - t_{n-1}\left(\frac{S}{\sqrt{n}}\right) < \mu < \bar{X} + t_{n-1}\left(\frac{S}{\sqrt{n}}\right)\right) = 1-\alpha$$

$$\rightarrow \left(74 - (2.353)\left(\frac{11.5}{\sqrt{4}}\right) < \mu < 74 + (2.353)\left(\frac{11.5}{\sqrt{4}}\right)\right)$$

$$\rightarrow (60.5 < \mu < 87.5)$$

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3- Confidence Interval for the difference in Two means ($\mu_1 - \mu_2$) when σ_1^2 and σ_2^2 are known.

$$\Rightarrow (\bar{X}_1 - \bar{X}_2) - Z_\alpha \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + Z_\alpha \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Example:- From 2 normally distributed samples x_1 is (64, 66, 89, 77) and x_2 is (56, 71, 53), if the Population variances for the 2 samples are both 96. Compute the 90% CI for the difference in means of the samples.

{Sol}:-

$x_1 \rightarrow (64, 66, 89, 77)$

$\bar{X}_1 = [64 + 66 + 89 + 77]/4 = 74$

$x_2 = (56, 71, 53) \rightarrow \bar{X}_2 = [56 + 71 + 53]/3 = 60$

→ from Table Z_α of 90% CI is 1.64

σ_1^2 and $\sigma_2^2 = 96$

$$\Rightarrow (\bar{X}_1 - \bar{X}_2) - Z_\alpha \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + Z_\alpha \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\Rightarrow (74 - 60) - 1.64 \sqrt{\frac{96}{4} + \frac{96}{3}} < \mu_1 - \mu_2 < (74 - 60) + 1.64 \sqrt{\frac{96}{4} + \frac{96}{3}}$$

$$1.73 < \mu_1 - \mu_2 < 26.27$$

The 90% CI for the difference in means is [1.73, 26.27]

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4- CI for the difference in Two means $(\mu_1-\mu_2)$ when σ_1^2 and σ_2^2 are Unknown.

$$\Rightarrow (\bar{X}_1-\bar{X}_2)-t\sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} < \mu_1-\mu_2 < (\bar{X}_1-\bar{X}_2)+t\sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

where

$$S_p^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$$

and the number of degrees of freedom is (n_1+n_2-2)

Ex) if we have $\bar{X}_1=74$, $\bar{X}_2=60$, $S_1^2=132.67$

$S_2^2=93.0$, $n_1=4$, $n_2=3$ Find 90% CI for $\mu_1-\mu_2$

Sol: $n_1=4$, $n_2=3$, $S_1^2=132.67$, $S_2^2=93.0$

$$S_p^2=\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}=\frac{(4-1)(132.67)+(3-1)(93.0)}{4+3-2}$$

$$\Rightarrow S_p^2=116.8$$

$\Rightarrow$ Degree of freedom $\Rightarrow (n_1+n_2-2)=5$

$\rightarrow t(5, 0.9)=2.015$

$$\Rightarrow (\bar{X}_1-\bar{X}_2)-t\sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} < \mu_1-\mu_2 < (\bar{X}_1-\bar{X}_2)+t\sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$