* Chapter Four *

∴ Interval Estimation Theory ∴

$$\Rightarrow (74-60) - 2.015\sqrt{116.8\left(\frac{1}{4}+\frac{1}{3}\right)} < \mu_1 - \mu_2 < (74-60) + 2.015\sqrt{116.8\left(\frac{1}{4}+\frac{1}{3}\right)}$$

$$\Rightarrow 14 - 16.63 < \mu_1 - \mu_2 < 14 + 16.63$$

$$\Rightarrow -2.63 < \mu_1 - \mu_2 < 30.63$$

5- CI for Population Variance (σ^2)

- let $X_1, X_2, \dots, X_n$ be a r.s of size n From $N(\mu, \sigma^2)$, Assume that the Two Parameters μ and σ^2 are Unknown, Then the $(1-\alpha)\%$ CI for the Pop var (σ^2) will be as:-

$$P\left[\frac{(n-1)S^2}{\chi^2\left(\frac{\alpha}{2}, n-1\right)} < \sigma^2 < \frac{(n-1)S^2}{\chi^2\left(1-\frac{\alpha}{2}, n-1\right)}\right] = 1-\alpha$$

where

$$(n-1)S^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}$$

Chapter Four

5- CI for Population Variance (σ^2)

Where

$$\rightarrow (n-1)S^2 = \sum_{i=1}^{n}(x_i-\bar{x})^2 = \sum_{i=1}^{n}x_i^2 - \frac{\left(\sum_{i=1}^{n}x_i\right)^2}{n}$$

Ex1) let $X_1, X_2, \ldots, X_8$ be a r.s. from $N(\mu,\sigma^2)$ where μ and σ^2 are unknown. $S^2 = 0.146$, Find 99% CI for σ^2.

Sol) $n=8$, $S^2 = 0.146$

$\rightarrow n-1=7 \quad \rightarrow (n-1)S^2 = 7.(0.146) = 1.02$

99% $\rightarrow$ From table $\quad 1-\alpha = 0.99$

$\alpha = 0.01$

$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

$1-\frac{\alpha}{2} = 0.995$

$\rightarrow \chi^2(\frac{\alpha}{2}, n-1) = \chi^2(0.005, 7) = 20.28$

$\chi^2(1-\frac{\alpha}{2}, n-1) = \chi^2(0.995, 7) = 0.989$

and by Using general Eq:-

$$P\left[\frac{(n-1)S^2}{\chi^2(\frac{\alpha}{2}, n-1)} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2(1-\frac{\alpha}{2}, n-1)}\right]$$

$$P\left[\frac{1.02}{20.28} \leq \sigma^2 \leq \frac{1.02}{0.989}\right] = 0.99$$

$$\rightarrow [0.05 \leq \sigma^2 \leq 1.03]$$

Chapter Four

CI for Population Variance (σ^2)

Ex2) let $x_1, x_2, \ldots, x_{10}$ be a r.s from $N(\mu, \sigma^2)$ where μ and σ^2 are unknown. Suppose $\sum_{i=1}^{10} x_i = 159$ and $\sum_{i=1}^{10} x_i^2 = 2531$ compute 95% CI for σ^2 where $\chi^2(0.025, 9) = 19.023$ and ~~$\chi^2(0.975, 9) = 2.70$~~

Sol) we have $(n-1)S^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2$

$$\rightarrow \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}$$

$$= 2531 - \frac{(159)^2}{10} = 2.9$$

$\rightarrow$ 95% CI $\rightarrow 1 - \alpha = 0.95 \rightarrow \alpha = 0.05$

$\rightarrow$ from table.

$\rightarrow \frac{\alpha}{2} = \frac{0.05}{2} = 0.025$ and $1 - \frac{\alpha}{2} = 0.975$

$\chi^2(0.025, 9) = 19.023$ and $\chi^2(0.975, 9) = 2.700$

by general Eq.

$$P\left[\frac{(n-1)S^2}{\chi^2(\frac{\alpha}{2}, n-1)} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2(1-\frac{\alpha}{2}, n-1)}\right] = 1 - \alpha$$

$$P\left[\frac{2.9}{19.023} \leq \sigma^2 \leq \frac{2.9}{2.700}\right] = 0.99$$

$$\rightarrow [0.152, 1.074]$$

Chapter Four
Interval Estimation Theory

6 - Confidence intervals for the binomial proportion parameter P.

- The CI for the proportion P (Probability for succeed in binomial Trials) can be given as:-

$$P - Z\sqrt{\frac{P(1-P)}{n}} < p < P + Z\sqrt{\frac{P(1-P)}{n}}$$

Ex: Consider a random sample of 144 families, 48 have 2 or more cars compute the 95% CI for the population proportion of families with 2 or mor cars.

$\boxed{\text{Sol}}$:- $n = 144$, $P = \frac{48}{144} = \frac{1}{3}$

From Table → 95% CI → $Z_\alpha = 1.96$

$1 - P = 1 - \frac{1}{3} = \frac{2}{3}$

↘ general Eq.

$$P - Z_\alpha\sqrt{\frac{P(1-P)}{n}} < p < P + Z_\alpha\sqrt{\frac{P(1-P)}{n}}$$

$$\frac{1}{3} - (1.96)\cdot\sqrt{\frac{\frac{1}{3}\cdot(\frac{2}{3})}{144}} < p < \frac{1}{3} + (1.96)\sqrt{\frac{(\frac{1}{3})(\frac{2}{3})}{144}}$$

$$0.333 - 0.077 < p < 0.333 + 0.077$$

$$0.256 < p < 0.410$$

* Interval Estimation theory *

Ex2) A random Sample of 100 People shows that 25 are left-handerd from a 95% CI for the True Proportion of left-handers.

Sol) $n = 100$ $\quad P = \frac{25}{100} = 0.25$

$1 - P = 1 - \frac{1}{4} = \frac{3}{4}$, $95\% \rightarrow Z_\alpha = 1.96$

by Using general Eq:-

$$P - Z_\alpha \sqrt{\frac{P(1-P)}{n}} < P < P + Z_\alpha \sqrt{\frac{P(1-P)}{n}}$$

$$\Rightarrow \frac{1}{4} - (1.96)\sqrt{\frac{\frac{1}{4} \cdot \left(\frac{3}{4}\right)}{100}} < P < \frac{1}{4} + (1.96)\sqrt{\frac{\frac{1}{4} \cdot \left(\frac{3}{4}\right)}{100}}$$

$$\Rightarrow 0.1651 < P < 0.3349$$

* Chapter Four *

Home Work:

1- let $X \sim N(\mu, 4)$, $n = 16$, $\bar{x} = 12$, Find 90% CI for μ.

2- let $X \sim N(\mu, \sigma^2)$, $n = 4$, $\bar{x} = 585.145$ and $S^2 = 0.01$, Find 99% CI for μ where $t(3, 0.99) = 4.541$

3- let $X \sim N(\mu, \sigma^2)$, $n = 16$, $\bar{x} = 11.4$ and $S^2 = 4.84$ Find 95% CI for μ. where $t(15, 0.95) = 1.753$

4- let $n = 15$, $p = 0.44$ From a 90% CI For the True Proportion P.

5- let $X_1, X_2, \ldots, X_{20}$ be a r.s. From $N(\mu, \sigma^2)$ where μ and σ^2 are unknown and $S = 1.6$ Compute 95% CI for σ^2.

Where $\chi^2(0.025, 19) = 32.852$
$\chi^2(0.975, 19) = 8.907$