**Number systems :**

In general any number N can be represented in the base ( radix ) R as show below :

 NR = dnRn  + dn-1 Rn-1 + ……… d1R1 + d0R0

**1-Decimal system ( R= 10 ) :**

This system uses ( 10 ) symbols ( 0 – 9 )

***Ex***: N10 = 23326180 = 2x103 + 3 x 102  +6x101 + 8x100 = 2000 + 300 + 60 + 8= 2368

**2-Binary system (R = 2 ) :**

It uses only two basic symbol ( 0,1 ) . It is the most suitable number system for digital circuits.

***Ex***: N2 = 1413020110 =1(2)4 + 1(2)3 + 0(2)2 + 0(2)1 + 1(2)0  = 16 + 8+0+0+1= ( 25 ) D

***Ex***: 13020110 = 1(2)3 + 0(2)2 + 0(2)1+ 1(2)0= 8+0+0+1 = (9)D

**3-Octal system ( R= 8 ) :**

It uses 8 symbols ( 0- 7 )

***Ex*** : 1170 = 1(8)1 + 7(8)0 = 8+7 =( 15)D

***Ex***: 73024150= 7(8)3 + o(8)2  + 4(8)1 +5(8)0 = 3584 +0 + 32 + 5=( 3621)D

**4- Hexadecimal system ( R =16 )**

It uses 16 symbol :

( 0,1,2,…………..,9,A,B,C,D,E,F )

***Ex***: 11A0 = 1(16)1+ A(16)0 = 16+10 =(26)D

 ***Ex***:329150 = 3(16)2+ 9(16)1 + 5 (16)0 =768 + 144 + 5 = ( 773)D

**The following table gives the correspondence between the four number system :**

|  |  |  |  |
| --- | --- | --- | --- |
| **Hexadecimal** | **Octal** | **Binary** | **Decimal** |
| 0 | 0 | 0 | 0 |
| 1 | 1 | 01 | 1 |
| 2 | 2 | 10 | 2 |
| 3 | 3 | 11 | 3 |
| 4 | 4 | 100 | 4 |
| 5 | 5 | 101 | 5 |
| 6 | 6 | 110 | 6 |
| 7 | 7 | 111 | 7 |
| 8 | 10 | 1000 | 8 |
| 9 | 11 | 1001 | 9 |
| A | 12 | 1010 | 10 |
| B | 13 | 1011 | 11 |
| C |  | 1100 | 12 |
| D |  |  |  |
| E |  |  |  |
| F |  |  |  |
| 10 |  |  |  |
| 11 |  |  |  |
| 12 |  |  |  |

Continue the above table.

**(Binary , octal and Hexadecimal) to Decimal :**

**For binary number** :

***Ex*** : ( 11010 . 101) 2 = 1(2)4+1(2)3+ 0(2)2+ 1(2)1+ 0(2)0+ 1(2)-1+ 0(2)-2+1(2)-3 =

 16+8+2+0.5+0.125 = ( 26.625)10

**For octal number :**

***Ex***: ( 613.24)8 = 6(8)2 +1(8)1+3(8)0+2(8)-1 + 4(8)-2 =

 384 + 8 + 3 + 0.25 + 0.0625 = ( 395.3125)10

**For hexadecimal :**

***Ex***: ( 5A.E)16 =5( 16)1+ A(16)0+ E(16)-1 =

 5(16)+10(16)+ 14(16)-1 = 80+10+0.875 = (90.875)10

**Decimal to (Binary , octal and Hexadecimal) :**

**1- To Binary :**

***Ex***: (353)10 = (?)2

 الباقي

 353 2 1 **LSB**

 176 2 0

 88 2 0

 44 2 0

 22 2 0

 11 2 1

 5 2 1

 2 2 0

 1 2 1 **MSB**

 0

 (353)10 = (101100001)2

***H.W.:*** (49302)10 = (?)2

 (10010)10 = (?)2

***EX***: (0.65625)10 = (?)2

 0.65625

 \_\_\_\_\_ 2 x

 1 1.31250

 **اول مرتبه بعد الفارزه**

 0.31250

 \_\_\_\_\_ 2 x

 0 0.62500

 \_\_\_\_\_ 2 x

 1 1.250

 2 x

 0 0.50

 2 x

 1 1.0

 (0.65625)10 = (0.10101)2

***H.W*.:** (0.8134)10 = (?)2

 (741.528)10 = (?)2

**2- to Octal :**

***EX*** : (254.75)10 = (?)8

  **الباقي**

 254 8 6 **LSB**

 31 8 7

 3 8 3

 0

 0.75

 8 x

 6 6.00

 (254.75)10 = (376.6)8

***H.W.* :** (3070.446)10 = (?)8

 (7706.77)10  = (?)8

**3- to Hexadecimal :**

***EX***: (567.1875)10 = (?)16

 567 16 7 **LSB**

 35 16 3

 2 16 2

 0

 0.1875

 16 x

3 3.0000

 (567.1875)10 = (237.3)16

***H.W.***(1005.36)10 = (?)16

 (3987.055)10 = (?)16

**Binary , Octal and Hexadecimal conversion(fast method) :**

***EX***:

 1 2 7 5 4 3 Octal

 1 0 1 0 1 1 1 1 0 1 1 0 0 0 1 1 Binary

  A  F  6  3  Hexadecimal

 =(44899)decimal how !

***EX:***

  2 1 4 4 3 5  Octal

 1 0 0 0 1 1 0 0 1 0 0 0 1 1 1 0 1 Binary

 1  1  9  1 D  Hexadecimal

***H.w:***

(1011100011)B =( ?)0 =(?)H

(AF31C)H= (?)O =(?)B

(37012)0= (?)B =(?)H

**Arithmetic operation :**

 1-Binary addition :

 Carry sum

 0 + 0 = 0

 0 + 1 = 1

 1 + 0 = 1

 1 + 1 = 1 0

***EX:***

 1 1 1

 11 100 111 110

 +11 + 10 + 11 +100

 110 110 1010 1010

**2-Binary subtraction :**

 0 – 0 = 0

 1 – 1 = 0

 1 – 0 = 1

  10 0 – 1 = 1 with borrow 1

***EX:***

  10 10

1. 11 101 110

 -01 -10 -011 -101

 10 01 010 001

**3-Binary multiplication :**

 0 x 0 = 0

 0 x 1 = 0 the same manner as in Decimal

 1 x 0 = 0

 1 x 1 = 1

***EX:***

 11 111

 X 11 x 101

 11 111

 + 11 + 000

 1001 111

100011

 **Hexadecimal addition :**

***EX:***

 23 58 2B

 +16 +22 +84

 39H 7AH  AFH

***EX:***  1

 D F 15D + 12D = 27D 27D -16D = 11D = BH with 1 carry

 +A C

 18 B

 1 + 13D + 10D = 24D 24D – 16D = 8D = 8H with 1 carry

**Octal addition :**

***EX:***

 14

 +23

 37o

***EX:***

 3 7 7O + 3O = 10D – 8D = 2D = 2O + 1 carry

 + 5 3

 11 2

 1 + 3O + 5O = 9D – 8D = 1D = 1O + 1 carry

**Complements :**

 Complements are used in digital computer for simplifying the subtraction operation and for logical manipulation . There are two types of complement for each base (R) system :

1-The R´s complement

2-the (R-1)´s complement

For binary number 1´s and 2´s complement

For decimal number 9´s and 10´s complement

For octal number 7's and 8's complement

For hexadecimal number 15's and 16's complement

**The 1´s and 2´s complement :**

 The 1´s complement of a binary number is the no. we get when we change each (0) to (1) and each (1) to (0) (or subtracting each binary no. from 1 )

***EX***: 1´s comp. of 1001 0110

 1´s comp. of 110010 001101

 2´s comp. = 1´s comp. + 1

 2´s comp. of 1011 is 0100 + 1 = 0101

 2´s comp. of 1110 is 0001 + 1= 0010

**Using 2´s complement in subtraction :**

 Instead of subtraction a numbe , we can add it’s$ $2's comp, and disregard the last carry.

***EX***: decimal

 7 111 111

 -5 -101 1´s 010 2´s 011

 2 1+ 1 010 + ve. No.

 X carry 011

***EX***: 13 1101 1101

 -10 1010 1´s 0101 2´s 0110

 3 1+ 1 0011 +ve. No.

 0110 X carry

***EX***: 4 100 100

 -7 -111 1´s 000 2´s 001+

 -3 1+ 101

 001 No carry -ve. No.

 So 101 100 011

**Using 1´s complement in subtraction :**

 Instead of subtracting a number we add the 1´s complement of the number , the last carry is then added to the number to get the final answer .

***EX***: 7 111 111

010 +

 -5 - 101 1´s

 2 carry 1 001

 + ve. No. 1 +

010

***EX***: 3 011 011

 -5 -101 1´s +010

 -2 101

 No carry -ve. No.

 101 010

**9´s and 10´s complements in decimal :**

 The 9´s comp. (similar to the 1´s comp.) is found by subtracting each decimal digit from 9 .

***EX***: 99

 - 25

 74 is the 9´s comp. of 25

***EX***: 9999

 -6291

 3708 is the 9´s comp.

10´s comp. = 9´s comp. + 1

***EX***: 74 + 1 =75 is the 10´s comp. of 25

***EX***: 9.99

 - 4.52

 5.47 is 9´s comp.

 + 1

 is 10´s comp 5.48

**7´s and 8´s complements in octal :**

 7´s = 7 – each digit

 8´s = 7´s + 1

***EX:*** 7777

 - 2415

 is 7´s comp. 5362

 +1

 5363 is 8´s comp.

***EX:*** Perform 75268 - 31428 using 8's comp.:

 7777

 -3142

 4635 + 1 = 4636

 1 1

 7526

 + 4636

  12 – 8 = 4

 11 - 8 = 3

 12 - 8 = 4

 1 4364 the result

***H.W.*** Perform the following using 8's complement:

 5458 – 148 =

 67768 – 43378 =

**15´s and 16´s complements in hexadecimal :**

 ***EX***: Find 15´s and 16´s comp. of ( 1 F A D )16

 15 15 15 15

 - 1 F A D

 E 0 5 2 15´s comp.

 1 +

 E 0 5 3 16´s comp.

 ***EX***: Perform A B E D – 1 F A D using 16's comp. :

  1 1

 A B E D

 + E 0 5 3

 16 – 16 = 0

 20 – 16 = 4

 24 – 16 = 8

 1 8 C 4 0 the result

 ***H.W.***:Perform the following using 16's complement :

 F E E D16 – D A F 316 = ANS: 23FA16

 9 8 A E16 – 1 F E E16 =

**4-Binary division :**

The standard division format is:

 Dividend =quotient

 Divisor

The divisor can be subtracted from the dividend a number of times equal to the quotient. For example in decimal:

21 21-7=14 1st

7

 14-7=7 2nd

 7-7=0 3rd so quotient= 3

In binary:

***EX:*** 1100

 100

100 0100 1's comp. 1011 2's comp. 1100

 1100

 1100 +

X 1 1000 q=1

 1100+

X 1 0100 q=2

 1100 +

X 1 0000 q=3 the result

***EX:*** Divide 1010 by 101:

101 0101 1's comp. 1010 2's comp. 1011

 1010

 1011 +

X 1 0101 q=1

 1011 +

X 1 0000 q=2 the result

**Boolean algebra :**

Boolean variable is a quantity that is either (1) or (0) .

 Boolean algebra is a mathematical method to manipulate Boolean variable in order to simplify the Boolean equation .

There are three basic operation in Boolean algebra :

1- OR (logical addition) (+)

2- AND (logical multiplication) (.)

3- NOT (logical inversion) ( )

**Gate :**

 It is a device with one output and two or more inputs . output is related to the input in Boolean (logic) equation .

 In the following , the basic logic gates with their symbols , truth table and equation :

 **Gate Symbol Truth table Equation**

 AB Z

 00 0

 01 1

 OR A Z 10 1 Z=A+B

 B 11 1

 AB Z

 00 0

 01 0

 AND A Z 10 0 Z=A.B

 B  11 1

 A Z

 0 1

 NOT A Z  1 0 Z=A

 AB Z

 00 1

 01 1

 NAND  A Z10 1 Z=A.B

 B  11 0

 AB Z

 00 1

 01 0

 NOR A  Z  10 0 Z=A+B

 B  11 0

 AB Z

 00 0

 01 1

 EX\_OR A  Z 10 1 Z=A + B

 B 11 0

 AB Z

 00 1

 01 0 Z=A + B

EX\_NOR A Z  10 0

 B 11 1

***EX***: Describe the T.T. for three input OR- gate :

 ABC Z

 000 0 A

 B Z  001 1

 C  010 1

 011 1

 100 1

 101 1

 110 1

 111 1

***EX***: Describe the T.T for the following logic circuit :

 A ABC Z

B  000 1

 Z  001 0

 C  010 1

 011 0

 100 1

 101 0

 110 1

 111 1

***H.w.***: Describe the T.T for the following :

A

A  B

B Z C Z

 c

 A A

 B B  Z

C Z

 D

**The basic rules and identities of Boolean algebra :**

 **1- Identities :**

 a- x.0=0 b- x+0=x c- x.1=x d- x+1=1

 e- x.x=x f- x+x=x g- x.x =0 h- x+x=1

 i- x=x

 **2-Commutative laws :**

 a- x.y=y.x

 b- x+y=y+x

 **3-Associative laws :**

 a- (x+y)+z = x+(y+z) = x+y+z

 b- (x.y).z = x.(y.z) = x.y.z

 **4-Distributive laws :**

 a- x.(y+z) = (x.y)+(x.z)

 b- x+(y.z) = (x+y).(x+Z)

 **5-Absorption laws :**

 a- x+x.y = x

 b- x+x.y = x+y

 c- x.(x+y) = x

 **6-De Morgan´s theorem :**

 a- x.y = x+y

 b- x+y = x.y

 All of these Boolean theorems useful in simplifying a logic expression that is in reducing the no. of terms in the expression .

***EX***: Prove that A.B = A+B

Sol: A.B = A+B = A+B

***EX***: Simplify the following :

 Z=A(A+B)

Sol: Z = AA+AB = A+AB = A

***EX***: write the Boolean exp. For the following circuit

 A

 B

 C Z

Sol: Z = (A+B).C

***EX***: write the Boolean exp. For the following logic cct.

 A

  Z

 B

Sol : Z = AB+AB = A + B

***EX***: write Boolean exp. For the following logic cct. :

 A

B

Z

C

Sol: Z = (AB+AB).C =(A + B).C

**NOTE :** The priorities of logic operation are :

 ( )

 AND

 OR

***EX***: Construct the logic circuit for the following Boolean exp. :

 Z = A.B+B

Sol:

 We need 2-input AND gate A

 B  2-input OR gate

 2-inverters Z

***EX***: Construct the logic cct. , and write the T.T. for the following logic eq.

 Z = AC+ABC

 ABC Z

Sol: 000 0

 A  001 0

 C  Z 010 0

 011 0

 B 100 0

 101 1

 110 0

 111 1

***Ex*:** simplify the following :

A= x y z + x y z + x y z + x y z =x z (y+y) + x z (y+y) = x z +x z = z (x+x) = z

***Ex:*** simplify the following :

Z= AB + A(B+C) + B(B+C)

 = AB + AB + AC + BB + BC

 = AB + AC + B + BC

 = AB + AC + B

 = B + AC

***Ex:*** Simplify the following :

Z = ABC + ABC + ABC

 = ABC + AC(B+B)

 = ABC + AC

 =C (AB+A)

 =C (A+B)

 =CA+CB

***Ex***: Simplify the following using Boolean Algebra

F= A{ BC(A+B+C+D)}

 = ABC(A+B+C+D)

 =ABCA +ABCB +ABCC +ABCD

 =ABC+ABC+ABC+ABCD

 =ABC+ABCD

 =ABC(1+D)

 = ABC

***EX***: Simplify:

F = AC + ABC + ACD + CD

 = A(C+BC) + C(AD+D)

 = A(C+B)+C(A+D)

 = AC + AB + CA +CD

 = A(C+C) + AB + CD

 = A + AB +CD

 = A(1+B) + CD

 = A + CD

***EX***: Prove that :

F= ABC + ABC + ABC = A(B+C)

 = AC(B+B) + ABC

 = AC + ABC

 = A(C+BC)

 = A(C+B)

***EX***: Simplify :

 X = AB + ABC + AB + ABC

 = AB(1+C) + AB + ABC

 = AB + AB + ABC

 = B + ABC

 = B + BAC

 = B + AC

***EX***: In a 3-input cct. The output is (1) if the majority of input is (1) , and otherwise , it is zero. White the T.T. for this cct. :

 ABC Z

 000 0

 001 0

 010 0

 011 1

 100 0

 101 1

 110 1

 111 1

**Sum -of- product representation of logic function :**

 A SP expression is a product term or several product terms, logically added together e.g:

 F= A.B + ABC + BD + …….

 product

 (AND)

**Derivation of sp :**

 1-construct the T.T.

 2-construct a multiplication column of product of all inputs.

 3-the desired expression is the sum of the product of all terms in which the output is 1 .

***EX***: For the following T.T. , write the logic function using sp method :

 P terms AB Z

 00 1 AB

 01 0 AB Z = AB + AB

 10 0 AB

 11 1 AB

***EX***: For the following T.T. , write the logic function using sp method , then simplify it :

 ABC Z P terms min terms

 000 0 ABC m0

 ABC m1 001 0

 m2 ABC 010 0

 011 1 ABC m3

 100 0 ABC m4

 101 1 ABC m5

 110 1 ABC m6

 111 1 ABC m7

 Z = m3 + m5 + m6 + m7

 = ABC + ABC +ABC + ABC

 = BC(A + A) + ABC + ABC

 = BC + ABC + ABC

 = C(B+BA) +ABC = C(B+A) +ABC

 = CB + CA + ABC = CB + A(C+BC)

 = CB + A(C+B)

 = CB + AC +AB

**Product -of- sum representation of logic function :**

 A PS is a sum term or several sum terms logically multiplied together e.g. :

F = (A+B)(A+B+C)(A+D)…..

**Derivation of PS :**

1-construct the T.T.

2-construct a sum column of sum of all inputs ( 0=uncomplement , 1=complement)

3-The desired output exp. Is the product of the sum of all terms in which the output

 is zero.

***EX:*** For the following T.T. , write the logic function using PS method :

 Z S. treams Max terms AB

 00 1 (A+B) M0

 01 0 (A+B) M1

 10 0 (A+B) M2

 11 0 (A+B) M3

Z= M1 . M2 . M3

 = (A+B)(A+B)(A+B)

***EX***: Simplify the following function using SP and PS methods :

 F(A,B,C) = π( M2 , M3 , M6 )

Sol:

 ABC Z

 000 1 m0

 001 1 m1

 010 0 M2

 011 0 M3

 100 1 m4

 101 1 m5

 110 0 M6

 111 1 m7

**1-By SP method :**

Z = m0 + m1 + m4 + m5 + m7

 = ABC + ABC + ABC +ABC + ABC

 = AB(C+C) + AB(C+C) + ABC  A

 = AB + AB + ABC C

 = B(A+A) + ABC B Z

 = B + BAC

Z = B + AC

**2- By PS method :**

Z = M2.M3.M6

 =(A+B+C)(A+B+C)(A+B+C)

 =(A+B+C)(A+B+C)(A+B+C)(A+B+C)

 =(AA+BA+CA+AB+BB+CB+AC+BC+CC)

 =(A+BA+CA+AB+B+CB+AC+BC).(

 =(A(1+B+C+B+C)+B(1+C+C)).(

 =(A+B)(AA+BA+CA+AB+BB+CB+AC+BC+CC)

 =(A+B)(BA+CA+AB+B+CB+AC+BC+C)

 =(A+B)(B(A+A+1+C+C)+C(A+A+1))

 =(A+B)(B+C)

  A

 B

 z

 C

PS method require one more gate than SP .

**Logic circuit design using NAND and NOR gates only :**

 There are many reasons for using NAND and NOR gates only to implement any logic function :

1-NAND and NOR gates are simpler, cheaper and have a faster response time to input changes , and consume less power .

2-The ability to implement any logic function using NAND or NOR gates only is easier than implement three different logic gates .

By using De morgan's theorem we can apply any logic circuit using NAND or NOR gates only.

**Using NAND gates :**

***EX***: Simplify the following function and implement the final equation using NAND gates only :

 H(A,B,C) = Σ (0,1,4,6,7)

 H= ABC + ABC + ABC + ABC + ABC

 = AB(C+C) + ABC + AB(C+C)

 = AB + ABC + AB

 = B(A+AC) + AB

 = B(A+C) + AB

 = BA + BC +AB

  B

  A

 B H

 C

 A

 B

H = H = (AB+BC+AB)

 = (A B) . (B C) . (AB)

 = X . Y . Z

Where: X = A B , Y = B C , Z = A B

  A

  B

 B  H = X . Y . Z

 C

 A

 B

This cct. is equivalent to the 1st one .

***EX***: Apply the following function using NAND gates only : F = A + BC

 Sol: F = F = (A B C) = (A) . (BC)

 = A . (BC)

 = X . Y

 Where X = A

 Y = BC

 A F

 B

 C

**Using NOR gates only :**

***EX***: Implement the following function using NOR gates only :

 G = (A+B) (A+C) (B+C)

 Sol: G = G = (A+B) (A+C) (B+C)

 = (A+B) + (A+C) + (B+C)

 = X + Y + Z

 Where X = (A+B) , Y = (A+C) , Z = (B+C)

  A X

  B

 A

C  Y G

 B Z

C

**Codes :**

 The binary number is the most natural system , but people are familiar to the decimal system . one way to solve this conflict is to convert all input decimal numbers into binary numbers and then convert the binary results back to decimal for the human user to understand . However , it is also possible for the computer to perform arithmetic operations directly with decimal numbers provided they are placed in registers in a coded form . When decimal numbers are used for internal arithmetic computations , they are converted to a binary code with four bits per digit .

It is very important to understand difference between the conversion of decimal numbers into binary and the binary coding for decimal numbers .

**BCD ( binary – coded – decimal ) :**

 These are codes that combine some of the features of both decimal and binary numbers.

There are different types of BCD codes :

**1- Excess-3 code :**

 It is an important BCD code . To encode decimal number to it’s excess-3 , we add (3) to each decimal digit before converting to binary :

|  |  |
| --- | --- |
| **ex-3****Note :** It is un weighted code .  | **Decimal** |
| 0011 | 0 |
| 0100 | 1 |
| 0101 | 2 |
| 0110 | 3 |
| 0111 | 4 |
| 1000 | 5 |
| 1001 | 6 |
| 1010 | 7 |
| 1011 | 8 |
| 1100 | 9 |
| 0100 0011 | 10 |
| 0100 0100 | 11 |

أكثر رقم نحتاجه هو 12

**2- BCD 8421 code :**

 It is weighted code .

|  |  |
| --- | --- |
| **BCD 8421** | **Decimal** |
| 0000 | 0 |
| 0001 | 1 |
| 0010 | 2 |
| 0011 | 3 |
| 0100 | 4 |
| 0101 | 5 |
| 0110 | 6 |
| 0111 | 7 |
| 1000 | 8 |
| 1001 | 9 |
| 0001 0000 | 10 |
| 0001 0001 | 11 |

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**3- Other 4-bit BCD codes :**

 Many other 4-bit codes exists , such as 7421 , 6311 , 5421 ,5311 , 5211 , 2421 …..

All are weighted codes .

***EX***: (16)D  to 2421 code

  1 6

 0001 1100

 (75)D  to 5421 code

 1010 1000

 (693)D to 6311 code

 1000 1100 0100

|  |  |  |  |
| --- | --- | --- | --- |
| **7421** | **6311** | **2421** | **D** |
| 0000 | 0000 | 0000 | 0 |
| 0001 | 0001 | 0001 | 1 |
| 0010 | 0011 | 0010 | 2 |
| 0011 | 0100 | 0011 | 3 |
| 0100 | 0101 | 0100 | 4 |
| 0101 | 0111 | 1011 | 5 |
| 0110 | 1000 | 1100 | 6 |
| 1000 | 1001 | 1101 | 7 |
| 1001 | 1011 | 1110 | 8 |
| 1010 | 1100 | 1111 | 9 |

|  |  |  |
| --- | --- | --- |
| **5311** | **5421** | **Decimal** |
| 0000 | 0000 | 0 |
| 0001 | 0001 | 1 |
| 0011 | 0010 | 2 |
| 0100 | 0011 | 3 |
| 0101 | 0100 | 4 |
| 1000 | 1000 | 5 |
| 1001 | 1001 | 6 |
| 1011 | 1010 | 7 |
| 1100 | 1011 | 8 |
| 1101 | 1100 | 9 |

**Gray code :**

 It is un weighted code the main characteristic of this code is that each gray number differs from the preceding number by single bit .

|  |  |  |
| --- | --- | --- |
| **Binary** | **Gray** | **Decimal** |
| 0000 | 0000 | 0 |
| 0001 | 0001 | 1 |
| 0010 | 0011 | 2 |
| 0011 | 0010 | 3 |
| 0100 | 0110 | 4 |
| 0101 | 0111 | 5 |
| 0110 | 0101 | 6 |
| 0111 | 0100 | 7 |
| 1000 | 1100 | 8 |
| 1001 | 1101 | 9 |
| 1010 | 1111 | 10 |
| 1011 | 1110 | 11 |
| 1100 | 1010 | 12 |
| 1101 | 1011 | 13 |
| 1110 | 1001 | 14 |
| 1111 | 1000 | 15 |

***EX***:

 1 0 1 0 Gray 1 1 0 0 Binary

 1 1 0 0 Binary 1 0 1 0 Gray

**Alpha numeric codes :**

 It is an assignment of bit combinations to the letters of the alpha bet , the decimal digit (0-9) , punctuation marks , and several special character such as # .

 The most widely used of alpha numeric codes are :

1- EBCDIC (Extended Binary Coded Decimal Interchange Code ).

 2- ASCII (American Standard Code for Information Interchange ).

 The EBCDK code uses 8-bit to represent each symbols while the ASCII code use 7-bit code.

**Parity method for error detection :**

 **Even parity(ep) :** makes the total no. of 1΄s even

 **Odd parity (op):** makes the total no. of 1΄s odd

|  |  |  |
| --- | --- | --- |
| **odd P** | **even P** | **number** |
| 1 | 0 | 0000 |
| 0 | 1 | 0001 |
| 0 | 1 | 0010 |
| 1 | 0 | 0011 |
| 0 | 1 | 0100 |
| 1 | 0 | 0101 |
| 1 | 0 | 0110 |
| 0 | 1 | 0111 |
| 0 | 1 | 1000 |
| 1 | 0 | 1001 |
| 1 | 0 | 1010 |
| 0 | 1 | 1011 |
| 1 | 0 | 1100 |
| 0 | 1 | 1101 |
| 0 | 1 | 1110 |
| 1 | 0 | 1111 |

***EX***: Check an even parity(ep) and odd parity(op) for the following numbers:

 0101, 0001

 For 0101 ep=0, op=1

For 0001 ep=1, op=0

**Karnaugh map simplification ( k-map ) :**

 It is an important method to simplify or minimize a Boolean expression . It is composed of number of adjacent "cells" . Each cell corresponds to a T.T. row , therefore there must be 2n

Cells in the k-map ( where n=no. of input variables ) .

**For two input variables (A&B) :**

B

B

B

B

A

 0 1

A

10

00

11

01

A

 0

 n=2 22 =4 cell

A

 1

**For three input variables ( A,B,C ) :**

BC

A

BC

 B C BC BC BC

 00 01 11 10

A

|  |  |  |  |
| --- | --- | --- | --- |
| AA |  |  |  |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| N=3 8 cell |  |  |  |
|  |  |  |  |

0

1

**For four input variables ( A,B,C,D ) :**

CD

CD

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| N=4 16 cell |  |  |  |
|  |  |  |  |
|  |  |  |  |

 C D CD CD CD

AB

 00 01 11 10

AB

A B

 AB

AB

AB

00

01

11

10

The first step in the minimization method is to implement the T.T. to the K-map.

1’s and 0’s in the output of the T.T. is placed in the cells corresponding to the input variables of the T.T.

A

B

0 1

 ***EX***: AB Z

 00 0

Z = B

0 0

0

1

 01 0

1 1

 10 1

 11 1

***Ex***: ABC Z

 000 0

 001 1

 010 0

 011 0

 100 0

 101 0

 110 1

 111 1

BC

A

0

1

10 11 01 00

|  |  |  |  |
| --- | --- | --- | --- |
| 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |

**Adjacent cells :**

The adjacent cells on k- map are those that differ by only one variable ( only one variable changes from 0 to 1 or 1 to 0 )

1 0

B

G1

A

|  |  |
| --- | --- |
| 1 | 001 |
| 1 | 0 |

 G1 = B

 00 01 11 10

BC

A

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 | 1 |  |  |

0

1

G1

 G1 = AB

 If more than one pair exist on k-map , we can OR the simplified products to get the final Boolean exp.

|  |  |  |  |
| --- | --- | --- | --- |
| 1 |  |  |  |
| 1 |  | 1 | 1  |

 A quad is a group of four 1’s that are horizontally or vertically adjacent . two variables are eliminated in the quad group.

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1 |  |  |
| 1 | 1 |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1 | 1 |  |
|  | 1 | 1 |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 | 1 | 1 | 1 |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  | 1 | 1 |  |
|  | 1 | 1 |  |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1 |  |  |
|  | 1 |  |  |
|  | 1 |  |  |
|  | 1 |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 | 1 | 1 | 1 |
|  |  |  |  |
|  |  |  |  |

 Octet is group of eight 1’s that are horizontally or vertically adjacent , so three variables can be eliminated .

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1 |  |  |
| 1 | 1 |  |  |
| 1 | 1 |  |  |
| 1 | 1 |  |  |

**Over lapping :**

 The same (1) can be used for more than one group

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  | 1 |  |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |

|  |  |  |  |
| --- | --- | --- | --- |
| 1 |  | 1 | 1 |
| 1 |  | 1 | 1 |
|  |  | 1 | 1 |
|  |  | 1 | 1 |

**Rolling :**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 |  |  | 1 |
| 1 |  |  | 1 |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| 1 |  |  | 1 |
|  |  |  |  |

**Summary of k-map method :**

 1-Implement the T.T. to k-map

 2-Encircle the octet , quads and pairs . Remember to roll and overlap to get the largest possible group.

 3- If any isolated 1’s , encircle each .

 4-Write the Boolean exp. By ORing the products corresponding to the encircle groups.

***EX***: simplify the following function using k-map

 F(A,B,C) = ∑ (0,1,2,3,5)

00 01 11 10

BC

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1 | 1 | 101 |
|  |  | 1 |  |

G1

A

G2

 F = G1 + G2

 = A + BC

***EX***: Simplify the following function using k-map

 F (ABC) = ∑ m0 , m1 , m4 , m5

BC

00 01 11 10

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | 1 | 101 |
|  |  | 1 | 1  |

A

G1

 F = G1

 = B

***EX***: Simplify the following function using k-map :

 F (ABCD) = ∑ (0,2,3,4,6,10,12,13,14)

AB

CD

00 01 11 10

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1 |  | 1 |
| 1 |  |  | 1 |
| 1 |  | 1 | 1 |
| 1 |  |  |  |

00

G4

G3

01

11

G1

G4

10

G2

 F = G1 + G2 + G3 + G4

***EX***: Find the simplified output in PS method using k-map for the following function :

 F = (ABC) = π (0,1,4,6)

00 01 11 10

BC

A

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | 0 | 0 |
| 0G2 |  |  | 0  |

01

G1

G2

 F = G1.G2

 = (A+B) (A+C)

***EX***: Simplify the following using k-map :

 F (ABCD) = π(0,1,2,3,5,7,8,9,10,11)

G2

AB

CD

00 01 11 10

|  |  |  |  |
| --- | --- | --- | --- |
| 0 | 0 | 0 | 0 |
|  | 0 | 0 |  |
|  |  |  |  |
| 0G1 | 0 | 0 | 0 |

G1

00

01

11

10

 F = G1 . G2

 = B(A+D)

**Don’t care condition :**

 Some logic ccts. can be designed so that there are certain input conditions for which there are no specified output levels , because these input conditions will never occur.

 It is necessary to specify the output for these conditions by either (0) or (1) in order to produce the simplest output exp.

***EX***: Simplify the following using k-map.

 F(ABCD) = ∑(0,3,6,15)

 dcc = 1,2,10,14

|  |  |  |  |
| --- | --- | --- | --- |
| d | 1 | d | 1 |
| 1G3 |  |  |  |
| d | 1 |  |  |
| dG2 |  |  |  |

AB

CD

00 01 11 10

G1

00

01

11

10

 F = G1 + G2 + G3

 = A B + CD + ABC

***EX***: Simplify the following function using k-map :

 F(ABCD) = π(5,6,7,13)

 dcc = (4,15)

CD

AB

00 01 11 10

|  |  |  |  |
| --- | --- | --- | --- |
| G1 |  |  |  |
| 0 | 0 | 0 | d |
| G2 | d | 0 |  |
|  |  |  |  |

00

01

11

10

 F = G1 . G2

 = (A+B) (B+D)

***H.W*.**: Simplify the following using PS and SP method by k-map :

 F(ABCD) = π(2,3,6,7,13)

 dcc = (0,4,8,9,10,12,14)

**5-variables k-map :**

n =5 32 cell

DE

DE

BC

00 01 11 10

BC

00 01 11 10

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

00

01

11

10

00

01

11

10

 A = 0 A = 1

 ملاحظة: توضع مجموعه فوق مجموعه بصورة متوازية أي أن ألا عمده الأفقية والعاموديه تتجاور مع مثيلاتها في ا المجموعة الثانية .

***EX***: Simplify the following logic function using k-map:

 F(ABCDE) = ∑ (0,1,4,5,10,16,17,20,21,26,30,31)

BC

BC

DE

00 01 11 10

00 01 11 10

DE

|  |  |  |  |
| --- | --- | --- | --- |
| G1 |  | 1 | 1 |
|  |  | 1 | 1 |
|  |  |  |  |
| 1 |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | 1 | 1 |
|  |  | 1 | 1 |
| 1 | 1 |  |  |
| 1 |  |  |  |

00

01

11

10

00

01

11

10

G1

G3

G2

G2

 A = 0 A = 1

 F = G1 + G2 + G3

 = B D + BCDE + ABCD

***EX***: Simplify the following , using k-map:

 F(ABCDE) = ∑ (0,1,2,3,6,15,24,25)

 dcc = (7,14,17,18,21,22,25,26,29,30)

BC

DE

00 01 11 10

BC

DE

00 01 11 10

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1 | 1 | 1 |
| 1 | d |  |  |
| dG1 | 1 |  |  |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| d | d |  |  |
| d | d |  |  |
| dG1 | d |  |  |
| d | d | 1 | 1 |

00

01

11

10

G2

00

01

11

10

G3

 A = 0 A = 1

 F = G1 + G2 + G3

 = CD + A B C + ABC

***H.W*.:** Simplify the following using k-map :

**Code converter :**

***EX***: design a logic cct. That convert BCD 8421 to EX-3 code :

 Sol:

8421 EX-3

ABCD WXYZ

0000 0011

0001 0100

0010 0101

0011 0110

Dcc = 10,11,12,13,14,15

0100 0111

0101 1000

0110 1001

0111 1010

1000 1011

1001 1100

CD

AB

00 01 11 10

AB

CD

00 01 11 10

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1W = | 1 | 1 |  |
| X | X | X | X |
| X | X | 1 | 1 |

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1 | 1 |  |
| X = |  |  | 1 |
| X | X | X | X |
| X | X | 1 |  |

00

01

11

10

00

01

11

10

CD

CD

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1 |  | 1 |
| Y = | 1 |  | 1 |
| X | X | X | X |
| X | X |  | 1 |

|  |  |  |  |
| --- | --- | --- | --- |
| 1 |  |  | 1 |
| 1Z = |  |  | 1 |
| X | X | X | X |
| X | X |  | 1 |

AB

00 01 11 10

AB

00 01 11 10

00

01

11

10

00

01

11

10

 ***EX***: Design a code converter cct. That convert BCD 6311 to BCD 5421 by using k-map :

ABCD WXYZ

0000 0000

0001 0001

0011 0010

Dcc = 2,6,10,13,14,15

0100 0011

0101 0100

0111 1000

1000 1001

1001 1010

1011 1011

1100 1100

00 01 11 10

|  |  |  |  |
| --- | --- | --- | --- |
| X |  |  |  |
| XX= |  | 1 |  |
| X | X | X | 1 |
| X |  |  |  |

CD

AB

CD

00 01 11 10

AB

|  |  |  |  |
| --- | --- | --- | --- |
| X |  |  |  |
| XW = | 1 |  |  |
| X | X | X | X |
| X | 1 | 1 | 1 |

00

01

11

10

00

01

11

10

|  |  |  |  |
| --- | --- | --- | --- |
| X | 1 |  |  |
| XY = |  |  | 1 |
| X | X | X |  |
| X | 1 | 1 |  |

|  |  |  |  |
| --- | --- | --- | --- |
| X |  | 1 |  |
| XZ= |  |  | 1 |
| X | X | X |  |
| X | 1 |  | 1 |

00

01

11

10

CD

AB

00 01 11 10

AB

CD

00

01

11

10

00 01 11 10

**Combinational circuits:**

A combinational circuit consists of logic gates whose output at any time are determined directly from the values of the present inputs.

Combinational

circuit

inputs outputs

**Design procedure:**

The procedure involves the following steps:

1-Determine the required number of inputs and outputs.

2-Drive the truth table.

3-Obtain the simplified Boolean function for each output.

4-Draw the logic diagram.

**Half adder (H.A.):**

To design a circuit that adds two numbers each of one bit for example:

 Two inputs 1 0 1 0

 1 + 0 + 0 + 1 +

 Two output 1 0 0 1 1

A B C S

0 0 0 0

0 1 0 1 S= A B + A B

1 0 0 1 C= A B

 1 1 1 0

 A B

 

S inputs C

 output

A S

 **H.A.** B C

**Using NAND gates only :**

S = A B + A B

S = A B + A B = A B . A B = X . Y



 C

 S

 A B

 A B A B

  C= C = A B = A B

**Full adder (F. A):**

في حالة جمع عددين كل واحد مكون من 2-bit أي لتصميم دائرة تجمع ال bits في المرحلة الثانية و ما بعدها:

Ci 1

A 0 1

A S

B F.A.

Ci Co

B 1 1 +

1 0 0

Carry sum

A B Ci Co S

0 0 0 0 0

0 0 1 0 1

0 1 0 0 1

0 1 1 1 0

1 0 0 0 1

1 0 1 1 0

1 1 0 1 0

1 1 1 1 1

S= A B Ci + A B Ci + A B Ci + A B Ci

S= Ci( A B + A B ) + Ci( A B + A B )

S= Ci ( A + B ) + Ci ( A + B )

Let A + B = K

 S= Ci K + Ci K = Ci + K

S= Ci + A + B

Co= A B Ci + A B Ci + A B Ci + A B Ci

Co= Ci ( AB + AB ) + AB ( Ci + Ci )

 Co= Ci . ( A + B ) + AB

 S

 Co



 A B Ci

**Using NAND gates only:**

S= A B Ci + A B Ci + A B Ci + A B Ci



 S

 Co=A B Ci + A B Ci + A B Ci + A B Ci

 

 Co

**Parallel binary adder:**

 لجمع رقمين كل منهما مكون من 2-bit

***Ex***: Design a logic circuit that add (3 + 1 )D :

 1 Ci

 1 1 A1 A0

 0 1 + B1 B0 +

1 0 0 Co S1 S0

 0 1 1 1 1

B0 A0

H.A.

Co S

B1 A1 Ci

F.A.

Co S

 1 0 1 0

**Half subtractor (H.S.):**

A B Bo D

0 0 0 0

0 1 1 1

1 0 0 1

1 1 0 0

D= A B + A B = A + B

Bo= A B

 A B

 D Bo



**Full subtractor ( F.S ):**

A B Bi Bo D

0 0 0 0 0

0 0 1 1 1

0 1 0 1 1

0 1 1 1 0

1 0 0 0 1

1 0 1 0 0

1 1 0 0 0

1 1 1 1 1

D=A B Bi + A B Bi + A B Bi + A B Bi

D= Bi ( A B + A B ) + Bi ( A B + A B )

D= Bi ( A + B ) + Bi ( A + B )

D= A + B + Bi

Bo= A B Bi + A B Bi + A B Bi + A B Bi

Bo= Bi ( A B + A B ) + A B ( Bi + Bi )

Bo= Bi ( A + B ) + A B

**Multiplexers (MUX):**

Multiplexer is a combinational circuit that selects one of many input lines and direct it to a single output line. The selection of a particular input line is controlled by a set of select variables. Normally there are ( 2n ) input lines and ( n) select variables whose bit combination determines which input is selected. The 4-to-1 multiplexer is shown below:

 S1 S0 Y

 0 0 D0

 0 1 D1

 1 0 D2

 So S1 D0 D1 D2 D3

 1 1 D3

 

 Y

There are: 2-to-1 MUX with 1 select variable

 4-to-1 MUX with 2 select variable.

8-to-1 MUX with 3 select variable.

. 16-to-1 MUX with 4 select variable

The circuit above can be implemented as an MSI chip, such a chip has four data inputs, two select variables and one output.

4-to-1 MUX

 D0

 D1 Y

 D2

 D3

 S1 S0

**Boolean function implementation using MUX :**

Boolean function of ( n )variables can be implemented with a multiplexer of either n, n-1, n-2…… select variables.

***Ex***: Implement the following function with (8-to1) and (4-to-1) MUX:

F(X,Y,Z)= ∑(1,2,6,7).

***First***: by using (8-t0-1) MUX:

 S2  S1 S0

 X Y Z F

 0 0 0 0 D0=0

 0 1

D0

D1

D2

D3 8-to-1

D4 Mux F

D5

D6

D7

 S2 S1 S0

 0 0 1 1 D1=1

 0 1 0 1 D2=1

 0 1 1 0 D3=0

 1 0 0 0 D4=0

 1 0 1 0 D5=0

 1 1 0 1 D6=1

 1 1 1 1 D7= 1

 X Y Z

***Second***: by using (4-t0-1) MUX:

X Y Z F

0 0 0 0

 Z Z 0 1

D0

D1

D2 4-to-1

D3 MUX

 S1 S0

0 0 1 1 D0=Z

0 1 0 1

O 1 1 0 D1=Z

1 0 0 0

1 1 0 1

1 1 1 1 D3=1

 X Y

***EX*:** Implement the following function using (8-to-1) and (4-to-1) MUX:

F(A,B,C,D)=∑( 1, 3, 4, 7, 11, 12, 13, 14, 15 ).

***First***: using (8-to-1) MUX:

A B C D F

 D D 0 1

0 0 0 0 0

0

1

2

3 8-to-1

4 MUx

5

6

7

 S2 S1 S0

O 0 0 1 1 D0= D

0 0 1 0 0

0 0 1 1 1 D1= D

0 1 0 0 1

0 1 0 1 0 D2= D

0 1 1 0 0

0 1 1 1 1 D3= D

1 0 0 0 0

1 0 0 1 0 D4=0

1 0 1 0 0

 A B C

1 0 1 1 1 D5= D

1 1 0 0 1

1 1 0 1 1 D6= 1

1 1 1 0 1

1 1 1 1 1 D7= 1

Using (4-to-1) MUX: ***second:***

A B C D F

0 0 0 0 0

O 0 0 1 1

0 0 1 0 0

0 0 1 1 1 D0= D

0 1 0 0 1

0 1 0 1 0

0 1 1 0 0

0 1 1 1 1 D1= C + D

1 0 0 0 0

1 0 0 1 0

1 0 1 0 0

1 0 1 1 1 D2= C . D

1 1 0 0 1

1 1 0 1 1

1 1 1 0 1

1 1 1 1 1 D3=

 D C

0

1 4-to-1 MUX

2

3 S1 S0

 1 

 F

 A B

**Demultiplexer:**

It is a digital circuit that performs the inverse operation of a multiplexer. It receives information from a single line and transmits it to one of 2n possible output lines. The selection of the specific output is controlled by the bit combination of (n ) select lines. A 4-to-1 DEMUX is shown below:

 E



 D0

 D1

 D2 D3

 S0 S1

For example when s1s0=10, output will be the same as the input value of E, while all the other outputs remain inactive at logic (0).

**Decoder :**

It is a combinational circuit that converts (n) inputs to a maximum of 2n unique outputs. A 2-to-4 decoder is shown below:



 A0 A1

 D0 =A1 A0 D1 = A1 A0 D2 = A1 A0 D3 = A1 A0

 A0 D0

 2-to-4 D1

 decoder D2

A1 D3

**Boolean function implementation using decoder :**

Any combinational circuit with ( n ) inputs and ( m ) outputs can be implemented with an n-to- 2n decoder and (m) OR gates. The Boolean function should be expressed in sum of product.

***Ex* :** Implement a full adder function with a decoder and OR gates:

From the truth table of full adder we get:

S(X,Y,Z) =∑ 1, 2, 4, 7

C(X,Y,Z) =∑ 3, 5, 6, 7

Since there are 3- inputs and a total of 8 minterms, then we need 3-to-8 decoder.

 Z X Y

 0

 1

 3-to-8 2

 decoder 3

 4

 5

 6

 7

 S C

**Encoder :**

It performs the inverse operation of a decoder. It has 2n (0r less) inputs and (n) output lines.

It is assumed that only one input has a value of (1) at any given time, otherwise the circuit has no meaning.

For example the 8-to-3 encoder has the following T.T. :

D7 D6 D5 D4 D3 D2 D1 D0 A2 A1 A0

 8-t0-3 encoder

0 0 0 0 0 0 1 0 0 0 1

 D0 D1  D2 A0  D3 A1 D4  A2 D5 D6 D7

0 0 0 0 0 1 0 0 0 1 0

0 0 0 0 1 0 0 0 0 1 1

0 0 0 1 0 0 0 0 1 0 0

0 0 1 0 0 0 0 0 1 0 1

0 1 0 0 0 0 0 0 1 1 0

1 0 0 0 0 0 0 0 1 1 1

**7- segment display :**

It consists of seven segments, usually LEDs or liquid crystals.

 a

 f g b

 e c

 d

we can display any decimal digit by turning on the appropriate elements (a…..g).

**BDC - T0 -7 segment decoder :**

It is a circuit with 4- bit input (BCD)and 7-outputs(segments). To display a number, the decoder must translate the input bits to the required output segment:

Digit D3 D2 D1 D0 a b c d e f g

0 0 0 0 0 1 1 1 1 1 1 0

1 0 0 0 1 0 1 1 0 0 0 0

2 0 0 1 0 1 1 0 1 1 0 1

3 0 0 1 1 1 1 1 1 0 0 1

4 0 1 0 0 0 1 1 0 0 1 1

5 0 1 0 1 1 0 1 1 0 1 1

6 0 1 1 0 0 0 1 1 1 1 1

7 0 1 1 1 1 1 1 0 0 0 0

8 1 0 0 0 1 1 1 1 1 1 1

9 1 0 0 1 1 1 1 0 0 1 1

BCD/7 segment decoder

D0 a

 D1 b

D2 c

D3 d

 e

 f

 g

BCD I/ps 7-seg.o/ps

**Digital comparator :**

It is a combinational circuit that compare two numbers A and B where each of them are 1-bit :

A B a :(A=B) b:(A$\ne $B) c:(A>B) d:(A<B)

0 0 1 0 0 0

0 1 0 1 0 1

1 0 0 1 1 0

1 1 1 0 0 0

a= A + B

1-1bit comparator

A a

B b

 c

 d

b= A + B

c= A B

d= A B

 A B

 

A

B

 b a c d

To compare 2-2 bit numbers:

A1 A0 =A

B1 B0=B

**A=B** when ( A1=B1 AND A0=B0 )

**A>B**  when ( A1>B1) OR (A1=B1 AND A0>B0 )

**A<B**  when (A1<B1) OR (A1=B1 AND A0<B0 )



 A1  B1 A0 B0

 X=Y X> Y

**Sequential circuits :**

The sequential circuits consists of a combinational circuit and storage elements as shown below:

Combinational

circuit

Inputs outputs

 next state

Storage

element

 present state

The next state of the storage elements is a function of the inputs and the present state.

There are two types of sequential circuit:

1-Asynchronous: The stored information in the stored element depends on the input signal only.

2-Synchronous: The stored information can change only during the occurrence of a clock pulse.

The storage elements are called flip-flop (f.f. ) . There are many types of f.f. :

**Set – Reset f.f. ( S-R f.f. ) :**

S Q

CK

R Q

 S CK R



 Q Q

CK S R Q Q

 0 X X X

 1 0 0 no change

 1 0 1 0 (reset) 1

1 1 0 1 (set) 0

1 1 1 not allowed

Note: Only when ck =1, the information from S and R is allowed to reach the output Q.

**D f.f. :**

The only way to eliminate the not *allowed state* in S-R ff is to ensure that both S & R will never be 1 at the same time. This is done in D-ff :

CK D Q Q

0 X X X

1 0 0 1

1 1 1 0

D Q

CK Q

S Q

Ck

R Q

 D

**J – K f.f. :**

 J K

S Q

R Q

\\

J Q

CK

K Q

CK J K Q

0 X X X

1 0 0 no change

1 0 1 0 (reset)

1 1 0 1(set)

1 1 1 toggle

**T – f.f. :**

CK T Q T

J Q

CK

K Q

T Q

CK Q

 0 x x

1 0 no change

1 1 toggle

***EX* :** Implement the following states on S-R ff. (initial state of QQ=10 )

**S R**  Q Q

**0 1** 0 1

**1 0** 1 0

 **1 1** not allowed

**0 0** 1 0

 **1 0** 1 0

***EX* :** Implement the following states on J-K ff (initial state of QQ=01)

J: 1 0 1 1 0 0

 K: 1 0 0 1 1 0

J k Q Q

1 1 1 0 toggle

0 0 1 0 no change

 1 0 1 0 set

toggle

 1 1 0 1

0 1 0 1 reset

0 0 0 1 no change

**Shift Register :**

It is a group of flip-flops that are capable of shifting binary information in one or both directions.

Shift registers are useful in:

1- Storage of serial data.

2- Serial to parallel or parallel to serial data conversion.

3- performing arithmetic operations

There are two types of data transfer:

**1- Serial transfer :**

Where data is transferred one bit at a time by shifting the bits of one ff into the next ff and so on.

CLR

 Serial output

D Q

D Q

D Q

D Q

Serial

input

CK

The serial input determines what goes into left most position during the shift. The serial output is taken from the output of the right most ff. the standard graphic symbol is:

 CLR

SRG4

 Serial input serial output

***EX*:** Shift the following data five pulses to the right:(**10111001**)

 CK

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

**Parallel transfer :**

Data can be transferred to or from all ffs at the same time:

 CLR

D Q

D Q

D Q

 CK

 I0  I1 I2

A universal register may perform different methods of moving data into or out of register like:

1- Parallel in – parallel out.

2- Parallel in – serial out.

3- Serial in – parallel out.

4- Serial in – serial out.

 CLR

A QA

B QB

C QC

D QD

RS

LS S1 S0

parallel load

 outputs

right serial input

left serial input modes of

 CK operation

**Counter :**

Flip-flops can be connected together to perform counting operations. The number of ffs used and the way in which they are connected determine the number of states (called modulus ).

Basically counters are of two types:

1- **Asynchronous counter ( Ripple counter ):**

An external clock signal is applied to the first ff and then the output of the preceding ff is connected to the clock of the next ff.

**EX:** Design 2- bit asynchronous binary counter:

Clk Q1 Q0 logic (1)

Initially 0 0

 1 0 1

2 1 0

J0 Q0

Ck

K0 Q0

J1 Q1

CK

K1 Q1

 3 1 1

4 0 0

**H.W. :**Design 3- bit asynchronous counter .

**BCD decade counters ( MOD 10 ) :**

The number of states are ten (0000 – 1001). In this type the counter should be count back to (0000) after (1001).

0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001

 Logic(1)

J0 Q0

CK

K0  CLR

J3 Q3

CK

K3 CLR

J2 Q2

CK

K2 CLR

J1 Q1

CK

K1 CLR

Skipped Q3 Q2 Q1 Q0

 1 0 1 0 0 1

 CLR

***EX*:** Design MOD 7 counter:

The counter should count from 000- 011 so there are 3 ffs. But state 111 should be skipped:

logic(1)

J0 Q0

CK

K0 CLR

J2  Q2

CK

K2 CLR

J1 Q1

CK

K1 CLR

**H.W. :**Design MOD 12 counter (0000 – 1011)

2- **Synchronous counter ( parallel counter ) :**

All the ffs in the counter are clocked at the same time by a common clock pulse.

 Logic(1)

***EX*:** Design 2-bit counter:

Clk Q1 Q0

Initially 0 0

J1 Q1

CK

K1  Q1

J0 Q0

CK

K0

1 0 1

2 1 0

3 1 1

 4 0 0

CLK

***EX*:** Design 3-bit synchronous counter:

 Logic(1)

CLK Q2 Q1 Q0

Initially 0 0 0

1 0 0 1

J1 Q1

CK

K1

J2 Q2

CK

K2

J0 Q0

CK

K0

2 0 1 0

3 0 1 1

4 1 0 0

CLK

5 1 0 1

**EX:** Design 4-bit Johnson counter:

CLK Q0 Q1 Q2 Q3

D2 Q2

 CK

D3 Q3

 CK Q3

0 0 0 0 0

D0 Q0

 CK

D1 Q1

 CK

1 1 0 0 0

2 1 1 0 0

3 1 1 1 0

CLK

4 1 1 1 1

5 0 1 1 1

6 0 0 1 1

7 0 0 0 1

**EX:** Design 4- bit Ring counter:

D3  Q3

 CK

0 1 0 0 0

D0 Q0

 CK

D1 Q1

 CK

D2 Q2

 CK

1 0 1 0 0

2 0 0 1 0

3 0 0 0 1 CLK

Note: initially a (1) present into first ff and the rest are cleared.

**ROM( read only memory) :**

A ROM contains permanently or semipermanently stored data, which can be read from the memory but either cannot be changed at all or cannot be changed without specialized equipment. ROMs retain stored data when the power is OFF.

**ROM family:**

ROM

UVPROM

Mask

ROM

PROM

EPROM

EEPROM

**A simple ROM :**

***EX***: Draw 16x8 ROM: 1 0

Address 0

Decoder 1

4-16 2

 3

 4

 .

 .

 16

 Address

Input

lines

 ……………………

 0 1 6 7

 data output lines (word)

ROMs can be used as look-up-tables (LUT) for code conversion and logic function.

***EX***: Show a ROM programmed for 4-bit binary to gray conversion:

 Binary Gray

A B C D W X Y Z

0 0 0 0 0 0 0 0

0 0 0 1 0 0 0 1

0 0 1 0 0 0 1 1

A ROM W

B X

C 16x4 Y

D Z

 . .

 . .

1 1 1 1 1 0 0 0

Address 0

Decoder 1

4-16 2

 3

 4

 .

 .

 16

 Address

Input

lines

 ……………………

 0 1 6 7

 data output lines (word)

**Memory expansion :**

Memory chip is basically as below:

***EX***: 16K x 1 ROM CS (chip select)

 A1

16K x 1

 . Data line

 .

 A14

***EX***: construct (64K x 16) memory using (16K x 1) memory chip:

64K = 216 16 address line ( A1………A16)

16K= 214 14 address line (A1………A14)

We need 64K= 4 row

 16K

We need 16 = 16 column

 1

 16- 14 = 2 ( A15 , A16 ) we use them in a 2- to-4 decoder to select rows:

 CS1 CS2 A15 ……………………………...

16K x 1

16K x 1

16K x 1

16K x 1

16K x 1

16K x 1

16K x 1

2-to-4

decoder

A16 CS3 ……………………………..

16K x 1

 CS4 …………………………………

16K x 1

***EX***: construct (4K x 4) memory using (1K x 4) memory chip.

4K = 4 row , 4 =1 column

1K 4

4K= 212

1K= 210 12-10=2 then we use 2-to-4 decoder

1K x 4

 A1

D1 . D4

 A10

 D1 . . D4  D1 . . D4 D1 . . D4

1K x 4

 A11 A12

2-to-4

decoder

1K x 4

1K x 4